Hypothesis Testing Guide

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Hypotheses – Null Hypothesis (H_0)

- A statement about a population parameter that is thought to be already true.
- We reject or fail to reject $\mathbf{H_0}$ based on statistical evidence.
- Usually $\mathbf{H_0}$ is a statement of ***no effect*** or ***no difference*** (=).

FOR EXAMPLE:

- *There no difference between means of y (output variable) values when level of x (input variable) changes from "a" to "b" ($\mu_a = \mu_b$) or,*
- *There no difference between variances of y (output variable) values when level of x (input variable) changes from "a" to "b" ($\sigma^2_a = \sigma^2_b$) or,*
- *There no difference between proportion of success of y (output variable) values when level of x (input variable) changes from "a" to "b" ($P_a = P_b$) or,*
- *…., or shortly*

Changing level of x does not effect y

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Hypotheses – Alternative Hypothesis (H_a)

- A statement about a population parameter that is suspected of being true, if $\mathbf{H_0}$ rejected.
- Needs evidence.
- Usually $\mathbf{H_a}$ is a statement of ***there is effect*** or ***there is difference*** (<,≠,>).

FOR EXAMPLE:

- *Mean value of y (output variable) decreases when level of x (input variable) changes from "a" to "b" ($\mu_a > \mu_b$), or*
- *Variance of y (output variable) increases when level of x (input variable) changes from "a" to "b" ($\sigma^2_a < \sigma^2_b$), or*
- *Proportion of success changes when level of x (input variable) changes from "a" to "b" ($P_a \neq P_b$), or*
- *…., or shortly*

Changing level of x effects y

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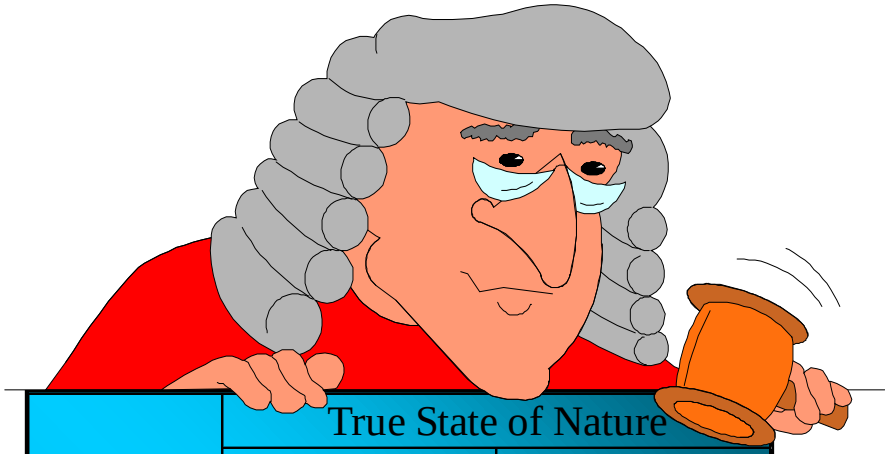
Order in the Court

- Hypothesis testing is like a contemporary legal system where a person is assumed innocent until proved guilty:
  - H_0: Defendant is innocent (Assumed)
  - H_a: Defendant is guilty (Must be proved)



		True State of Nature	
		Did Not Commit Crime	Commited A Crime
Verdict	Did Not Commit Crime	**Correct Verdict**	**Guilty Goes Free**
	Commited A Crime	**Innocent Person Convicted**	**Correct Verdict**

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Hypothesis Testing Results

- Type I Error
  - Rejecting the null hypothesis when it is, in fact, true.
  - The probability of a Type I error is denoted by α ($0< \alpha <1$).
  - α is also known as producer or supplier risk.
- Type II Error
  - Accepting the null hypothesis, when it is, in fact, false.
  - The probability of a Type II error is denoted by β ($0< \beta <1$).
  - β is also known as consumer or customer risk.

		True State of Nature	
		Ho	**Ha**
Conclusion Drawn	**Ho**	**Correct Decision**	**Type II Error**
	Ha	**Type I Error**	**Correct Decision**

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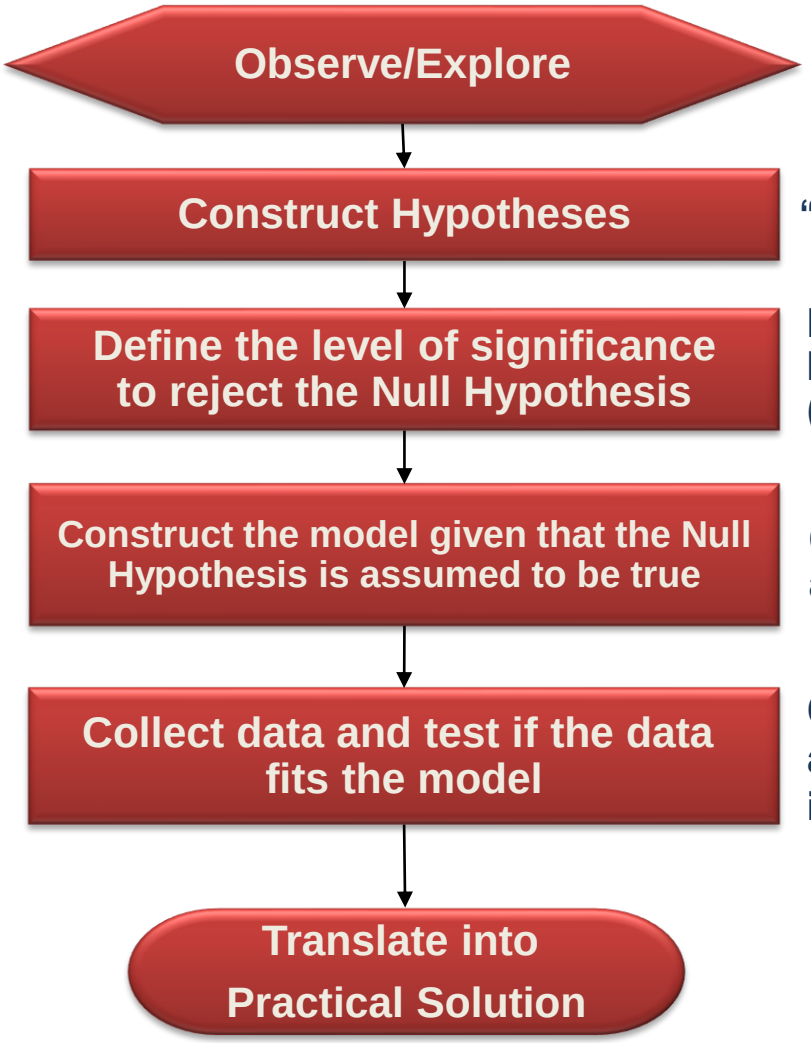
The Essence of Hypotesis Testing



"Null Hypothesis" and "Alternative Hypothesis"

Define measure of the strength of the evidence that must be present in the data for rejecting the "Null Hypothesis" (We reject Null hypothesis if we see ….)

Construct the mathematical model for the population assuming Null Hypothesis is true.

Calculate "Significance Value" (the probability of obtaining a result at least as extreme, given that the null hypothesis is true) – Compare it by the "Significance Level".

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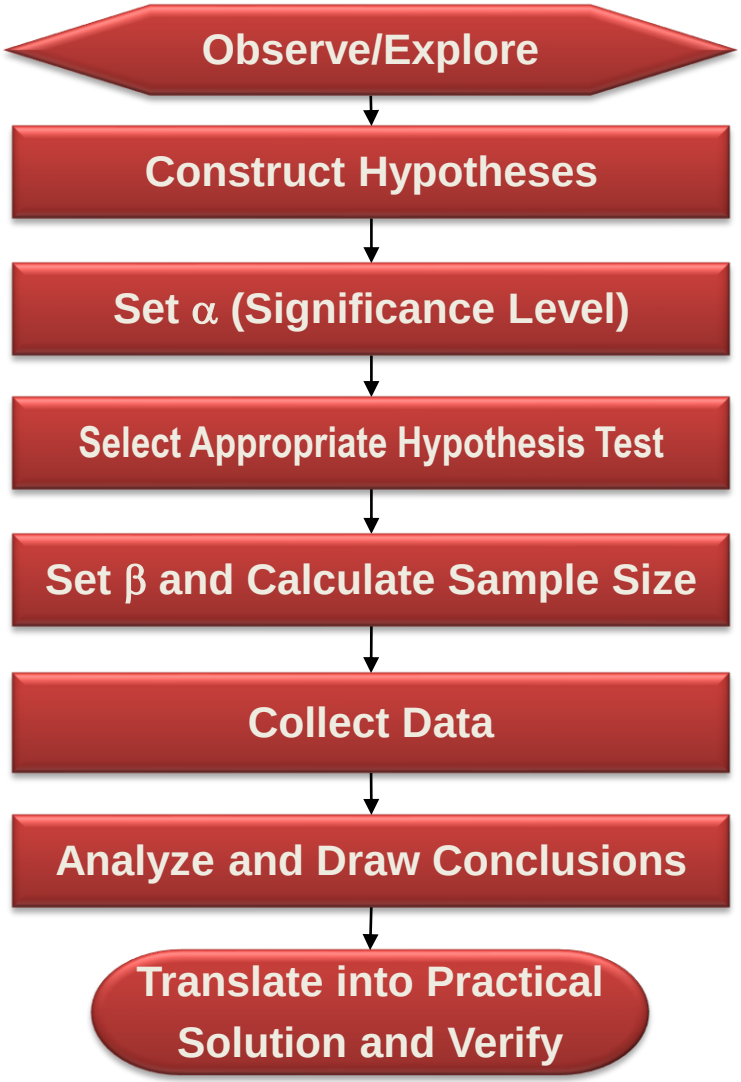
Hypothesis Testing Recipe



H_0: There is no difference (=)
H_a: There is difference (<, ≠, >)

Generally 0.05 (%5)

HYPOTHESIS TESTING ROAD MAP

β generally is 0.10 (%10)

Random, independent, calculated amount

Calculate "Significance Value"
(P-Value: the probability of obtaining a result at least as extreme, given that the null hypothesis is true)
– Compare it by The "Significance Level".

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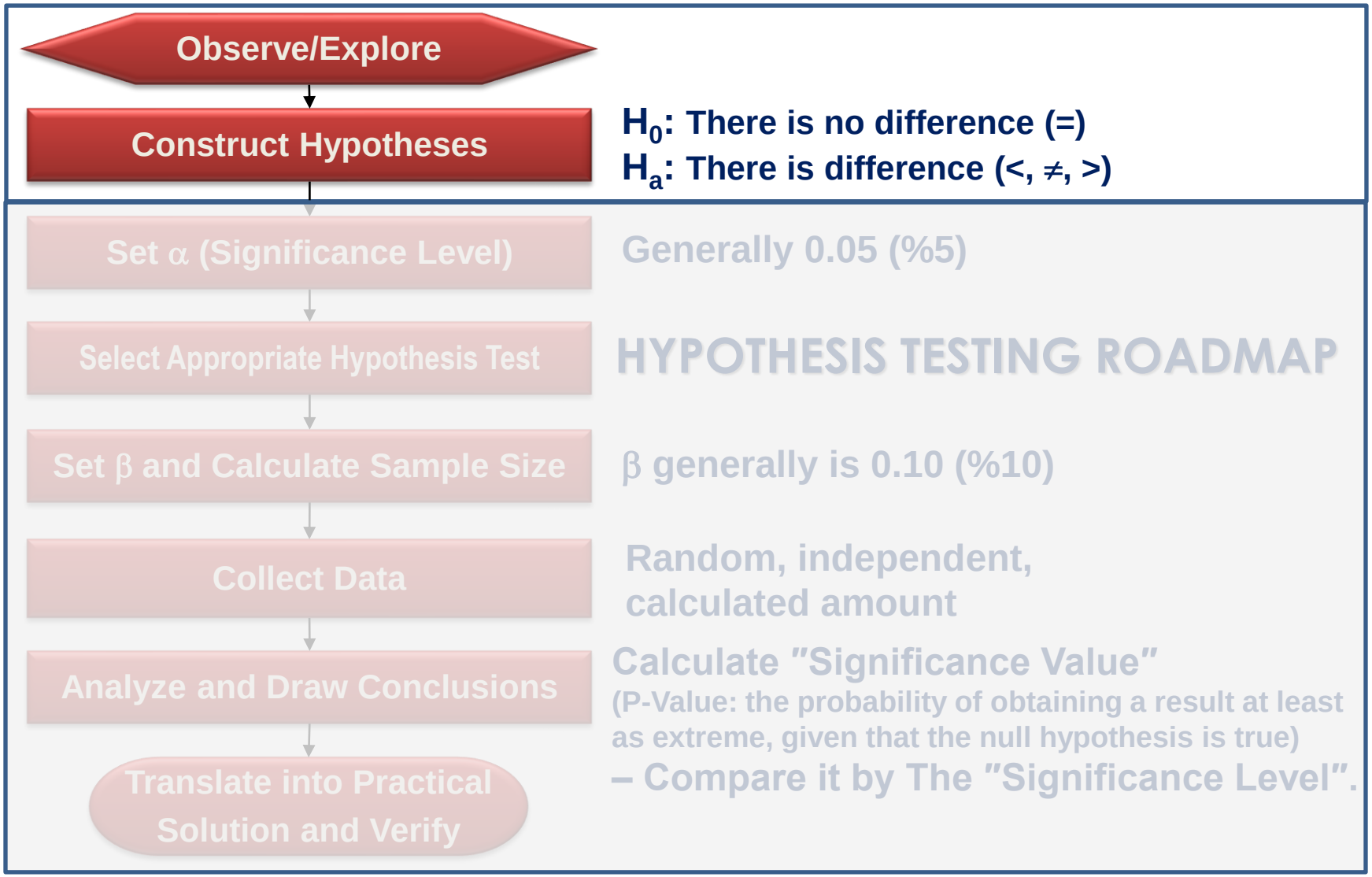
Hypothesis Testing Recipe



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Constructing Hypotheses

1. Practical problem: (What is the decision to be made?)
2. -y: What? How is it measured (data type)? Measurement system?

   -x: What (What do we think effects y)?

   - Levels of x?
3. Hypotheses:
   - H_0: (=) *Changing the level of x does not effect y.*
   - H_a: (<,≠,>) *Changing the level of x effects y.*

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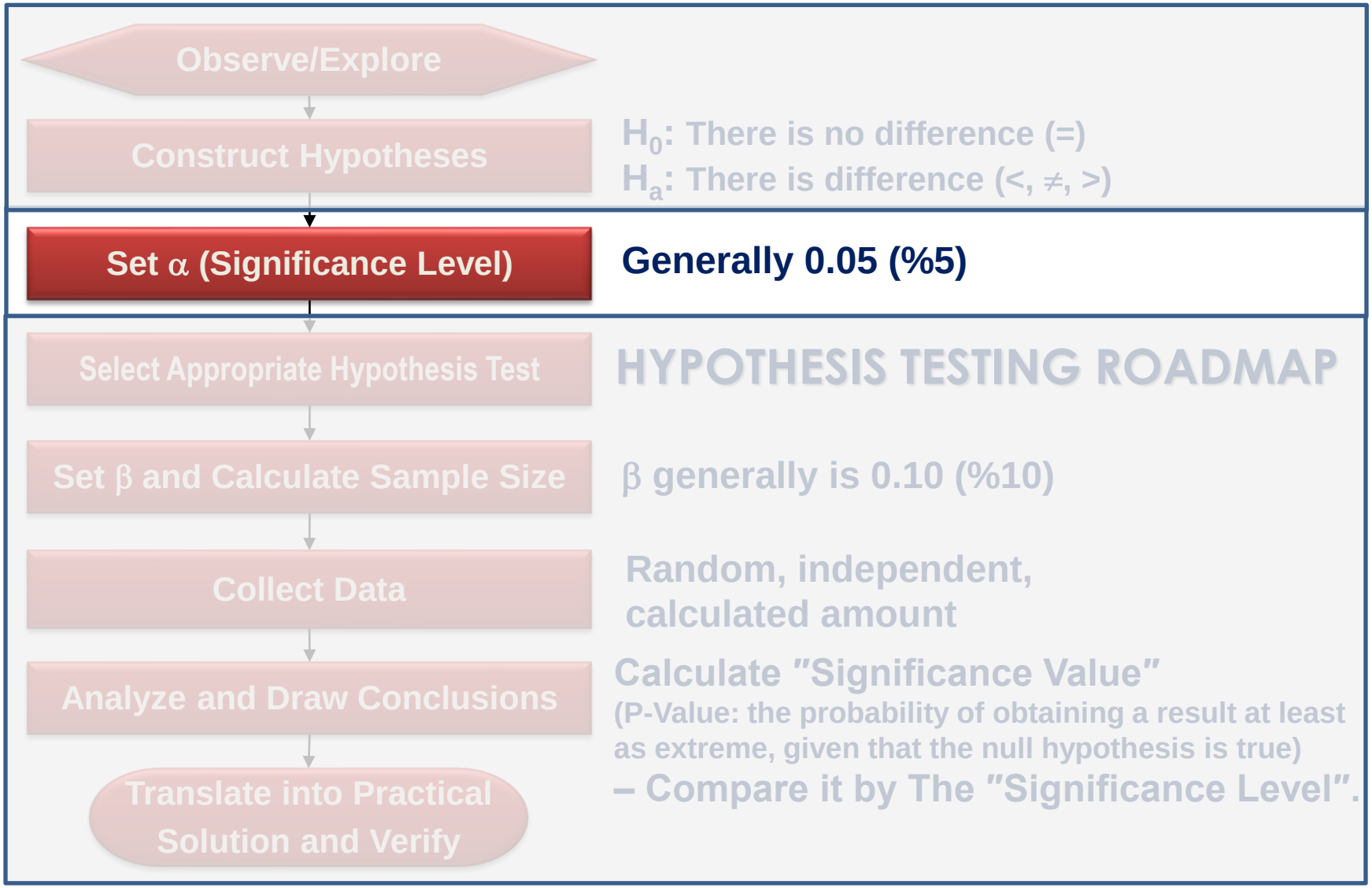
Hypothesis Testing Recipe



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Significance Level: α (Risk of Type I Error)

- The probability of rejecting the null hypothesis (H_0) when it is, in fact, true: α
  - Define α (define probality of making type I error).
- Define measure of the strength of the evidence that must be present in the data for rejecting the "Null Hypothesis" (We reject Null hypothesis if we see …).

If the characteristic under consideration is not a safety characteristic than generally:

α = 0,05 (%5)

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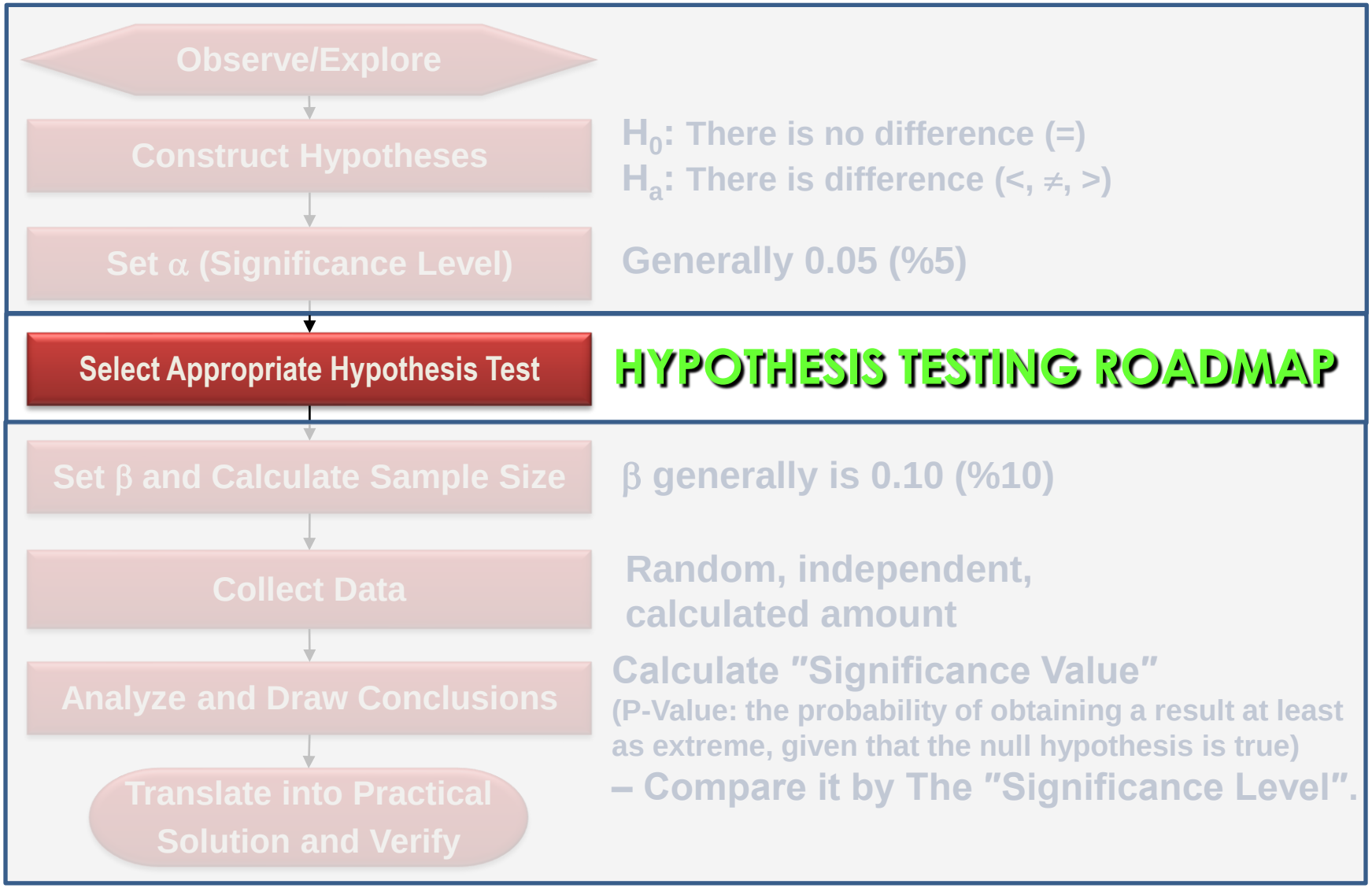
Hypothesis Testing Recipe



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Hypothesis Testing Roadmap

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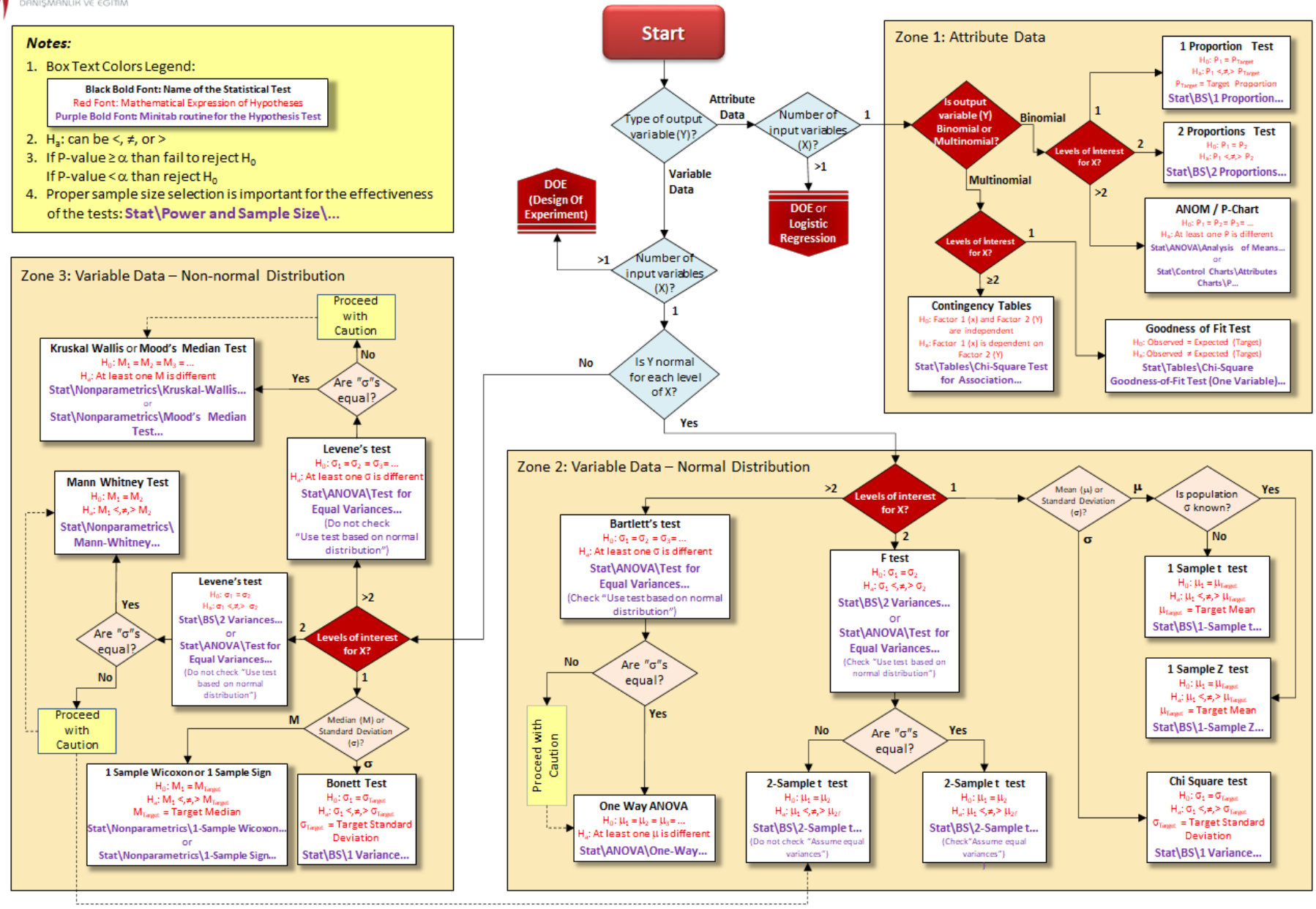
DANIŞMANLIK VE EĞİTİM

Notes:

1. Box Text Colors Legend:
   - **Black Bold Font: Name of the Statistical Test**
   - Red Font: Mathematical Expression of Hypotheses
   - Purple Bold Font: Minitab routine for the Hypothesis Test
2. H_a: can be <, ≠, or >
3. If P-value ≥ α than fail to reject H_0
   If P-value < α than reject H_0
4. Proper sample size selection is important for the effectiveness of the tests: **Stat\Power and Sample Size\...**



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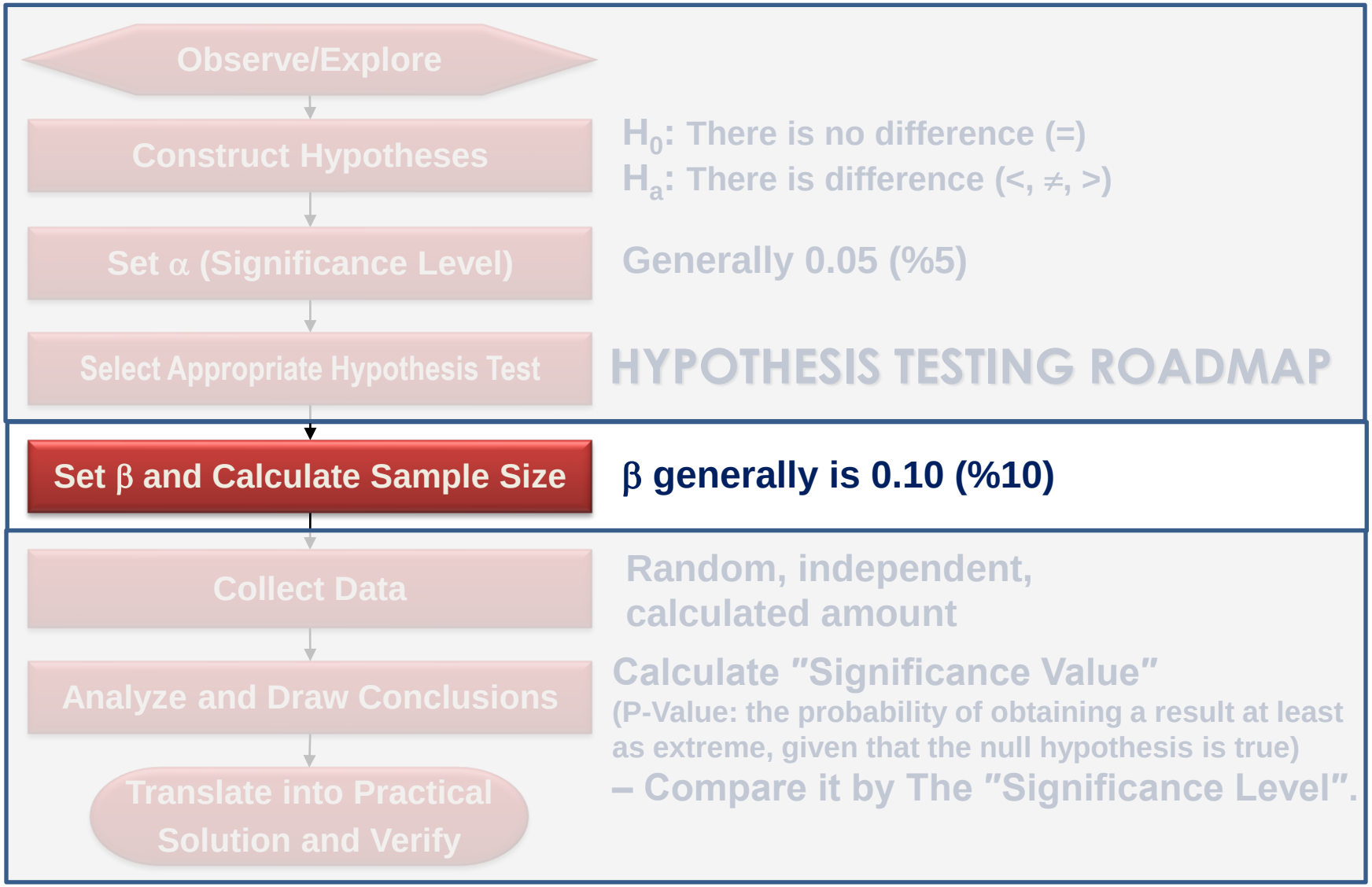
Hypothesis Testing Recipe



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Sample Size: $n = f(\delta, \beta, \alpha, Test, H_a)$

- δ: Critical Difference: What is the difference you want to detect?
  - The minimum practical difference that is of financial or technical consequence to your business.
- β: the possibility of missing a difference of size δ.
  - β: The probability (risk) of a Type II Error.
  - Accepting the null hypothesis, when it is, in fact, false.

or

- Power: The ability to detect a critical difference (δ). The probability (chance) of detecting δ.
  - Power = 1 – β

If the characteristic under consideration is not a safety characteristic than generally:

β = 0.10 (%10)

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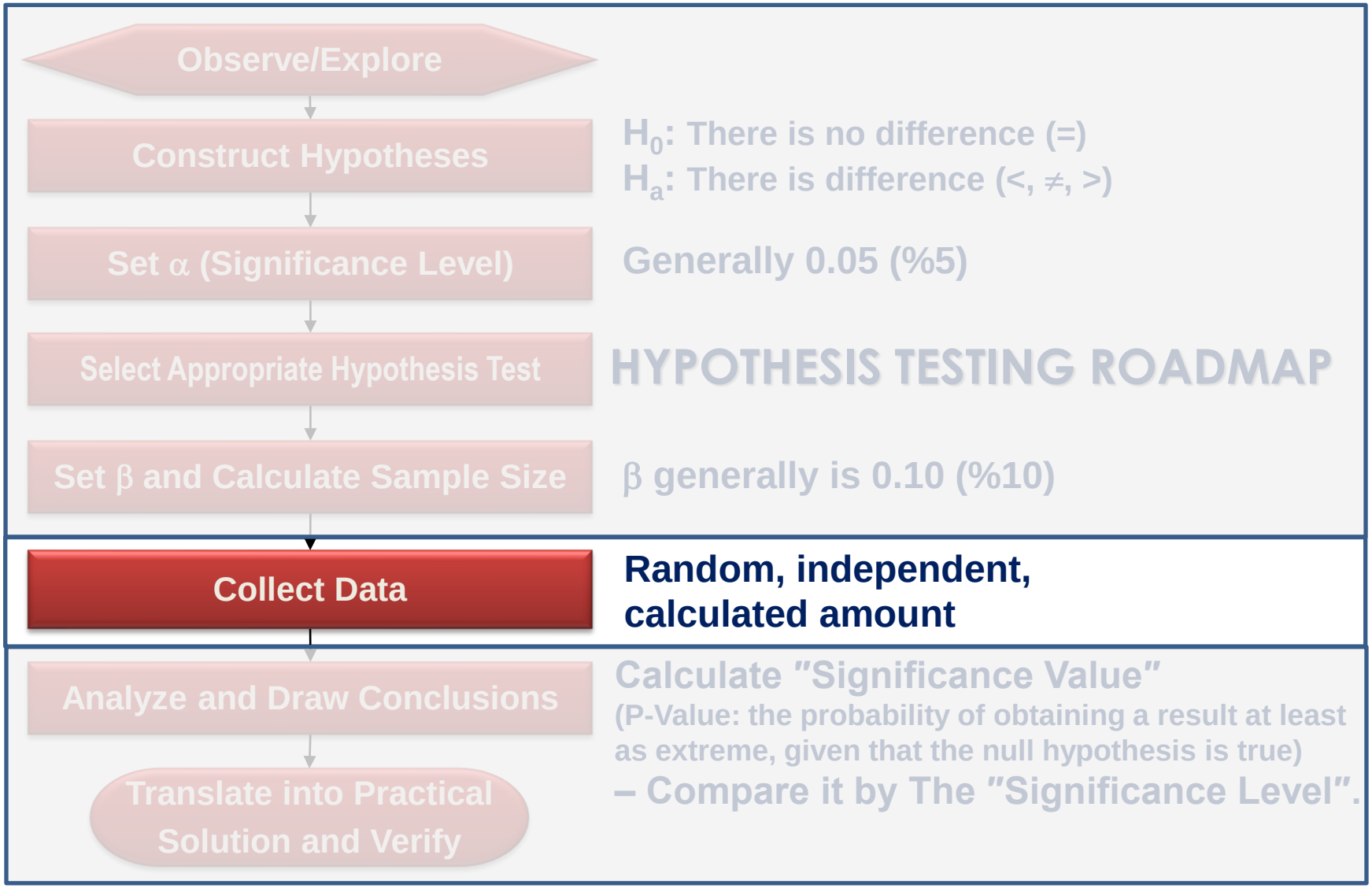
Hypothesis Testing Recipe



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Collect the Data

- Collect the data
  - Random
  - Independent
  - Calculated amount

CAUTION:

- Be sure that measurement system is capable.

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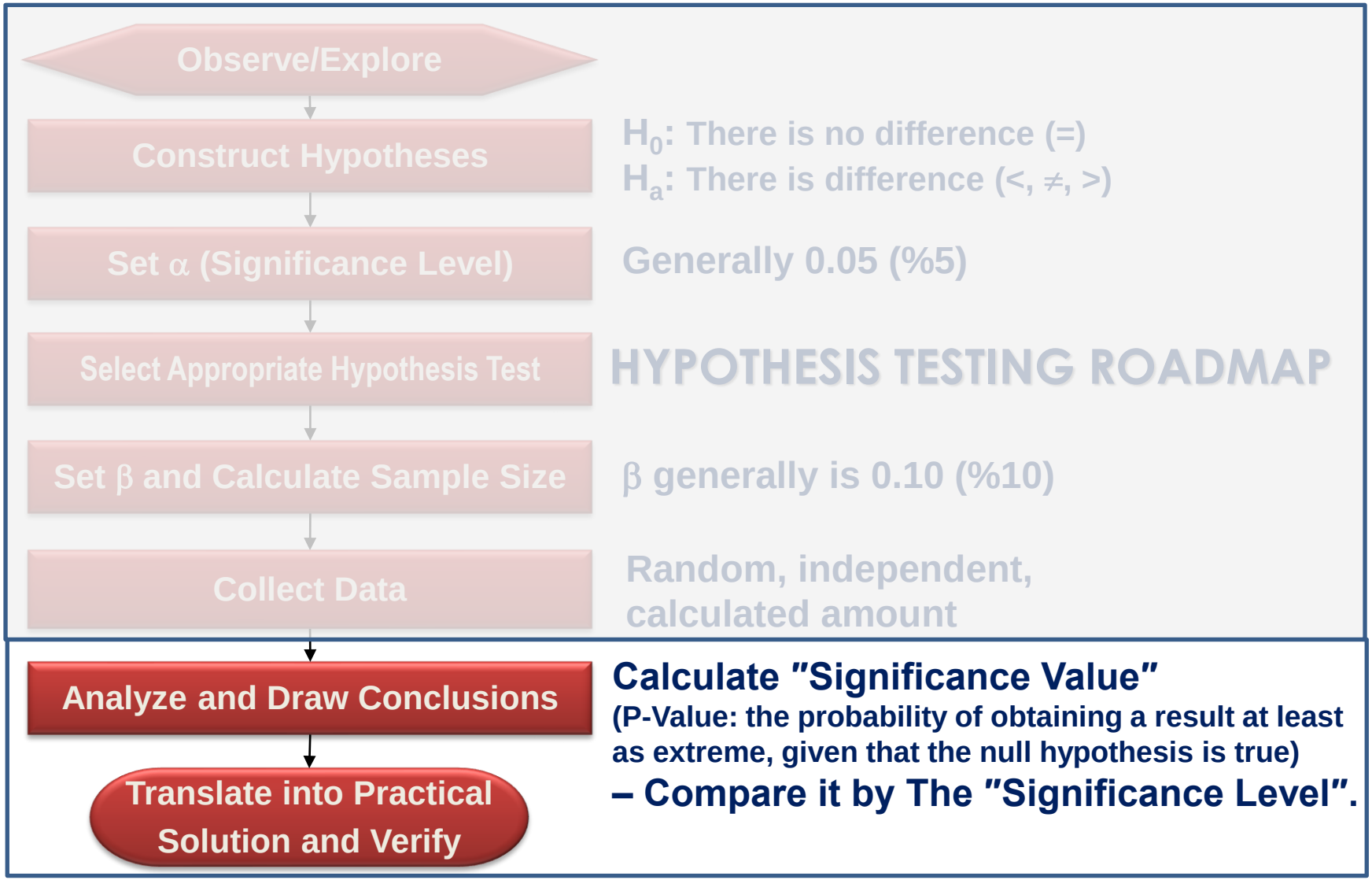
Hypothesis Testing Recipe



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Analyze and Draw Conclusions

- Calculate significance value (P-Value) and reject Null Hypothesis if it is less than significance level (α).

If P-Value < α than reject H_0

- Translate the statistical conclusion into practical conclusion.
- **Decide next step**

P-Value: The probability of obtaining a result at least as extreme, given that the null hypothesis (H_0) is true.

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